EXCERPTS FROM *CHEMISTRY WITHOUT EQUATIONS:*

A guide to the solution of chemistry problems using dimensional analysis

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Getting Started:

To solve a chemistry problem you need to answer the following questions:

- What am I looking for?
- What do I have?
- What more do I need?
- How do I put it all together?

What *are* your looking for?

In reading the problem, what you are *looking for* is the object of a phrase such as "How many…" or "Calculate the number of…." For example, in the problem "How many miles are there in 10.0 kilometers?" you are looking for the number of *miles*. There is only *one answer* (with units) to each problem, and that provides us with a method to solve problems: find the answer *units* and you find the answer!

What *do* you have?

What you *have* are the data in the problem and their attached units. Sometimes these are referred to as the "givens." In the problem "How many miles are there in 10.0 kilometers?" you are given *10.0 kilometers*. In other problems there may be more than one given. Sometimes some of the givens are not needed to solve the problem, and you must learn to recognize them and discard them.

What *more* do you need?

What you need in the problem "How many miles are there in 10.0 kilometers?" are *bridge factors* with units—either equalities or proportionalities—that allow you to convert from miles to kilometers. One or more bridging factors may be needed

What are *equalities*?

Equalities are the relationships between two identical quantities measured in two different unit systems. For example, "2.54 centimeters equals one inch." Mathematically this is written "2.54 centimeters = 1 inch." It may also be stated in others ways, such as "2.54 centimeters *in* one inch" or "2.54 centimeters *per* one inch." Obviously, the converse of these are also true, so "1 pound = 453.6 grams." We will write these as fractions, with the line separating the numerator (top) and the denominator (bottom) read "equals," "in," or "per."

$$\frac{2.54\text{ cm}}{1\text{ in.}} \quad \text{and} \quad \frac{1\text{ in.}}{2.54\text{ cm}}$$

Because the numerator and denominator represent the *identical* length in two different unit systems, the fractions both equal "1."

What are *proportionalities*?

Proportionalities are used the same as equalities in calculations, but they represent two quantities that are related to each other by a fraction or a multiple rather than one-to-one. For example, if every bag of M&M candies contains 36 candies, and six of them are red, we can write "6 red M&Ms $\propto$ 1 bag M&Ms" where the symbol "$\propto$" is read "proportional to." As with the equal sign, we can also read the proportionality sign as "in" or "per." Thus, the following fractions are all true:

$$\text{Equality:} \quad \frac{36\text{ M\&Ms}}{1\text{ bag M\&Ms}} \quad \text{and} \quad \frac{1\text{ bag M\&Ms}}{36\text{ M\&Ms}}$$

$$\text{Proportionality:} \quad \frac{6\text{ red M\&Ms}}{1\text{ bag M\&Ms}} \quad \text{and} \quad \frac{1\text{ bag M\&Ms}}{6\text{ red M\&Ms}}$$

How do you put it all together?

Putting the "given" together with the equalities or proportionalities into a problem solution requires that you analyze where you are and where you want to get to—a process called **dimensional analysis**. In this case it will be combined with the recognized problem solving technique of **working backwards** from the desired answer to the starting point.

Consider the following problem: At a gathering you meet a person who speaks only Chinese who is an expert in a field of interest to you. How do you speak with her if you do not speak Chinese? At the same gathering is a Dutch person who speaks English, German and French (English = German = French), A Korean who speaks Chinese (Korean = Chinese), a German who speaks French (German = French), and a Vietnamese person who speaks both French and Korean (French = Korean). These other people are able to provide you with language BRIDGES to get across the chasm that you have identified. Presuming that a minimum of number of translations would be most expedient, you might approach the problem in the following manner:

Since you know your "answer" is **Chinese**, place the Korean person, who will hear Chinese, first in line (the numerator of the first box). What is heard in Chinese will be equal to what the translator speaks in Korean (the denominator of the first box). The next person (Vietnamese) will hear Korean (the second numerator) and speak French (the second denominator). The next person (Dutch) will hear French (third numerator) and speak English (third denominator). You are then in a position to hear English. In terms of dimensional analysis the set-up looks like this:

$$\left|\frac{\textbf{Chinese}}{\cancel{\text{Korean}}}\right|\frac{\cancel{\text{Korean}}}{\cancel{\text{French}}}\left|\frac{\cancel{\text{French}}}{\cancel{\text{English}}}\right|\frac{\cancel{\text{English}}}{}\right| = \text{Chinese}$$

(Korean)(Vietnamese)(Dutch) (You)

Notice that *what you want* is on top ("up") in the first box, or FACTOR, and that the bottom ("down") is whatever is on the opposite side of the language EQUALITY—here Chinese = Korean, so Chinese is on top, Korean underneath. From then on, what is down in one factor is up in the next, like dominoes. Notice, too that all numerators and denominators "cancel" except for the thing you are looking for—in this case "Chinese."

Now consider the following numerical examples of problem solving by dimensional analysis:

EXAMPLE 1—USE OF PROPORTIONALITY:

How many bags of M&Ms candies must you buy to collect 60 red M&Ms candies?

From the wording of the problem you can see that you want "bag(s) of M&Ms." So the factor we want first in the problem must have these units in the numerator. Clearly this is the *left* factor of the two above. The solution is:

$$\left|\frac{1\ \textbf{bag}}{6\ \cancel{\text{red M\&Ms}}}\right|\frac{60\ \cancel{\text{red M\&Ms}}}{}\right| = 10\ \textbf{bags}\text{ of M\&Ms}$$

(1SF) (1SF) (1SF)

EXAMPLE 2—USE OF EQUALITY: TIME CONVERSIONS:

Wording #1: *Find the number of seconds in 0.025 hours*. (Worded as a command.)
Wording #2: *How many seconds are there in 0.025 hours?* (Worded as a question.)

A little thought will tell you (1) that you are looking for units of "seconds" (the key words are "how many" in the second wording) and (2) that in order to get from seconds to hours you need to pass through units of minutes. The equalities needed are those that BRIDGE the units seconds, minutes and hours. They are, of course:

60 seconds = 1 minute and 60 minutes = 1 hour

From these we get the following possible factors:

$$\frac{60\text{ seconds}}{1\text{ minute}} \quad\text{and}\quad \frac{1\text{ minute}}{60\text{ seconds}} \quad\text{and}\quad \frac{60\text{ minutes}}{1\text{ hour}} \quad\text{and}\quad \frac{1\text{ hour}}{60\text{ minutes}}$$

BRIDGES: SECONDS <=> MINUTES MINUTES <=> HOURS

Because the answer units you want are "seconds" they must be on the right *and* in the first numerator. The equality with "seconds" in it has "60 seconds," so that must be the first numerator, and the other side of that equality "1 minute" must be the first denominator—the first factor above. The second numerator must have the same units as the first denominator (minutes), so it must be "60 minutes" and the second denominator must be the other side of the second equality "1 hour"—the third factor above. The third numerator must have the units "hours" to match the second denominator so you use the "0.025 hours" you were given in the problem. The canceled units are "down-up" across the set-up:

$$\left|\frac{60\ \textbf{seconds}}{1\ \cancel{\text{minute}}}\right|\frac{60\ \cancel{\text{minutes}}}{1\ \cancel{\text{hour}}}\left|\frac{0.025\ \cancel{\text{hours}}}{}\right| = 9.0\ \textbf{seconds}$$

(exact) (exact) (2SF) (2SF)

EXAMPLE 3—CONVERSION: ENGLISH-TO-METRIC LENGTH:

Convert 6.2 miles to units of kilometers. (A runner may know the answer to this problem immediately.)

Before you start, make sure you have all the bridges needed to convert from kilometers to miles. Start with the desired units, kilometers (km) and do two metric conversions through the base unit (meters) while aiming for the units within the metric system, centimeters (cm), that allows conversion into the English system, inches (in), using the bridge, 2.54 cm = 1 in. The rest of the problem (from the fourth factor to the end) is simply an English units conversion set-up from inches to miles.

$$\left|\frac{1\ \mathbf{km}}{1000\ \text{m}}\right|\frac{1\ \text{m}}{100\ \text{cm}}\left|\frac{2.54\ \text{cm}}{1\ \text{in}}\right|\frac{12\ \text{in}}{1\ \text{ft}}\left|\frac{5280\ \text{ft}}{1\ \text{mi}}\right|\frac{6.2\ \text{mi}}{}\right| = 1.0\times10^{1}\ \mathbf{km}$$ (a "10K" race)

(exact) (exact) (exact) (exact) (exact) (<u>2SF</u>) (<u>2SF</u>)

EXAMPLE 4—CONVERSIONS USING CUBED UNITS:

How many cubic centimeters (cm^3) are there in 3.6 cubic yard (yd^3) ?

Initially ignore the cubed units. Set up the problem as a conversion between centimeters and yards.

$$\left|\frac{2.54\ \mathbf{cm}}{1\ \text{in}}\right|\frac{12\ \text{in}}{1\ \text{ft}}\left|\frac{3\ \text{ft}}{1\ \text{yd}}\right|\frac{3.6\ \text{yd}^3}{}\right| = \_\_\_\_\_\ \mathbf{cm}$$

Cube all the factors, but do not cube the 3.6 because the units are already cubed (yd^3).

$$\left|\frac{2.54^3\ \text{cm}^3}{1^3\ \text{in}^3}\right|\frac{12^3\ \text{in}^3}{1\ \text{ft}^3}\left|\frac{3^3\ \text{ft}^3}{1^3\ \text{yd}^3}\right|\frac{3.6\ \text{yd}^3}{}\right| = \_\_\_\_\_\ \mathbf{cm^3}$$

Cubing the numerical coefficients and units results in the final equation:

$$\left|\frac{16.39\ \mathbf{cm^3}}{1^3\ \text{in}^3}\right|\frac{1728\ \text{in}^3}{1\ \text{ft}^3}\left|\frac{27\ \text{ft}^3}{1^3\ \text{yd}^3}\right|\frac{3.6\ \text{yd}^3}{}\right| = 2.7\times10^{6}\ \mathbf{cm^3}$$

(exact) (exact) (exact) (<u>2SF</u>) (<u>2SF</u>)

EXAMPLE 5—SIMPLE DENSITY CALCULATION:

What is the density, in units of g/cm^3, of an object with a mass of 1.97 g and a volume of 0.781 cm^3?

To obtain the desired units of g/cm^3 it is necessary to place the given mass of 1.97 grams in the numerator of the first box and then to multiply this quantity by the inverse of the given volume of 0.781 cm^3, which places the desired units of volume (cm^3) in the denominator.

$$\left|\frac{1.97\ \mathbf{g}}{}\right|\frac{1}{0.781\ \mathbf{cm^3}}\right| = 2.52\ \frac{\text{g}}{\text{cm}^3} = 2.52\ \mathbf{g/cm^3}$$

(<u>3SF</u>) (<u>3SF</u>) (<u>3SF</u>)

EXAMPLE 6—METRIC-TO-ENGLISH DENSITY CONVERSION:

A liquid has a density of 1.60 g/mL. *Convert this density to units of pounds per gallon.*

The desired answer units are lb/gal. Cubic-length volume can be avoided here because the relationship "946 mL = 1 quart" allows conversation directly between English fluid-volume units and metric fluid-volume units.

$$\left|\frac{1\ \mathbf{lb}}{453.6\ \text{g}}\right|\frac{4\ \text{qts}}{1\ \mathbf{gal}}\left|\frac{1.60\ \text{g}}{\text{mL}}\right|\frac{946\ \text{mL}}{1\ \text{qt}}\right| = 13.3\ \frac{\mathbf{lb}}{\mathbf{gal}}$$

(4SF) (exact) (<u>3SF</u>) (<u>3SF</u>) (<u>3SF</u>)

EXAMPLE 7—CALCULATION OF A SPECIFIC HEAT FROM EXPERIMENTAL DATA:

If the input of 16.7 calories into 6.17 grams of a metal raises its temperature from 25.0°C to 50.0°C what is the specific heat of the metal in units of cal/g•C°?

Note: The temperature change, Δt_C = (50.0°C - 25.0°C) = 25.0 C°, becomes the third denominator.

$$\left|\frac{16.7\ \mathbf{cal}}{}\right|\frac{1}{6.17\ \mathbf{g}}\left|\frac{1}{25.0\ \mathbf{C^\circ}}\right| = 0.108\ \mathbf{cal/g{\bullet}C^\circ}$$

(<u>3SF</u>) (<u>3SF</u>) (<u>3SF</u>) (<u>3SF</u>)

EXAMPLE 8—CONVERSION OF METRIC SPECIFIC HEAT UNITS TO S.I. UNITS:

Convert the specific heat of iron, 0.106 cal/g•C°, to S.I. units of J/kg•K°.

$$\left|\frac{4.1840\ \mathbf{J}}{1\ \text{cal}}\right|\frac{1000\ \text{g}}{1\ \mathbf{kg}}\left|\frac{1\ \text{C}^\circ}{1\ \mathbf{K^\circ}}\right|\frac{0.106\ \text{cal}}{\text{g}{\bullet}\text{C}^\circ}\right| = 444\ \mathbf{J/kg{\bullet}K^\circ} = 4.44\times10^{2}\ \mathbf{J/kg{\bullet}K^\circ}$$

(exact) (exact) (exact) (<u>3SF</u>) (<u>3SF</u>) (<u>3SF</u>)

Notice that the required units (J/kg•K°) are constructed with the first numerator and the second and third denominators.

EXAMPLE 9—AVOGADRO'S NUMBER: GRAM-MOLE-ATOM CONVERSIONS

How many atoms of sodium (Na) are contained in a 10.0-gram sample of $Na_2OsO_2Cl_4$? The molar mass of $Na_2OsO_2Cl_4$ is 410.0 grams/mole.

This more complicated problem requires that you convert between the sub-microscopic (atomic) and macroscopic levels (using Avogadro's number), and between moles and grams of $Na_2OsO_2Cl_4$. Note that the formula shows that there are two moles atoms of sodium in one mole of compound. The solution begins with Avogadro's Number, then refers to the number of Na atoms in the formula, then to the molar mass:

$$\left|\frac{6.022\infty 10^{23}\ \mathbf{atoms\ Na}}{1\ \text{mol Na}}\right|\frac{2\ \text{mol Na}}{1\ \text{mol } Na_2OsO_2Cl_4}\left|\frac{1\ \text{mol } Na_2OsO_2Cl_4}{410.0\ \text{grams } Na_2OsO_2Cl_4}\right|\frac{10.0\ \text{grams } Na_2OsO_2Cl_4}{}\left| = 2.94\times 10^{22}\ \mathbf{atoms\ Na}\right.$$

(4SF) (exact) (4SF) (<u>3SF</u>) (<u>3SF</u>)

EXAMPLE 10—STOICHIOMETRIC CALCULATION OF REQUIRED AMOUNT OF REACTANT:

In the production of ammonia, how many grams of hydrogen are consumed in a ratio with excess nitrogen if 7.23 grams of ammonia are produced? (Reference the reaction equation in the previous example.)

Because the problem refers to "grams" of both H_2 and NH_3 the molar masses of both must be used in the solution.
The desired units "g H_2" are introduced into the calculation by the molar mass factor for H_2. The stoichiometric factor linking H_2 and NH_3 (from the balanced reaction equation) allows the cancellation of "mol H_2" and introduces "mol NH_3" which is then canceled by the "mol NH_3" in the molar mass factor for NH_3—the denominator of which cancels the given grams of ammonia. The solution is:

$$\left|\frac{2.016\ \mathbf{g\ H_2}}{1\ \text{mol } H_2}\right|\frac{3\ \text{mol } H_2}{2\ \text{mol } NH_3}\left|\frac{1\ \text{mol } NH_3}{17.03\ \text{g } NH_3}\right|\frac{7.23\ \text{g } NH_3}{}\left| = 1.28\ \text{g } \mathbf{H_2}\right.$$

(4SF) (exact) (4SF) (<u>3SF</u>) (<u>3SF</u>)

EXAMPLE 11—IDEAL GAS LAW: CALCULATE GAS MOLES, GIVEN PRESSURE, TEMP. AND VOLUME:

How many moles of a gas occupy a 2.50 L container at a pressure of 685 mm Hg and a temperature of 298K? (R = 0.08206 L•atm/mol•K)

Ideal gas behavior is assumed. The solution is then begun with R *inverted* so that desired "mol" are on top. The "atm" units are canceled using the relationship "1 atm = 760 mm Hg" (exact).

$$\left|\frac{\text{mol}\bullet\text{K}}{0.08206\ \text{L}\bullet\text{atm}}\right|\frac{2.50\ \text{L}}{298\text{K}}\left|\frac{1\ \text{atm}}{760\ \text{mm Hg}}\right|\frac{685\ \text{mm Hg}}{}\left| = 9.21\times 10^{-2}\ \mathbf{mol}\right.\ \text{(3SF)}$$

(4SF) (<u>3SF</u>/<u>3SF</u>)) (exact) (<u>3SF</u>) (<u>3SF</u>)

EXAMPLE 12—MOLARITY: VOLUME OF CONCENTRATED REAGENT REQUIRED FOR GIVEN DILUTION:

How many milliliters of 12.0 M *(conc.) HCl solution are required to prepare 5.00×10^2 mL of 2.00* M *(dilute) HCl solution?*

$$\left|\frac{1000\ \mathbf{mL\ conc}}{\text{L conc}}\right|\frac{1\ \text{L conc}}{12.0\ \text{mol HCl}}\left|\frac{2.00\ \text{mol HCl}}{1\ \text{L dil}}\right|\frac{1\ \text{L dil}}{1000\ \text{mL dil}}\left|\frac{5.00\infty 10^{2}\ \text{mL dil}}{}\right| = 83.3\ \mathbf{mL\ conc.}\ \text{HCl (3SF)}$$

(exact) (<u>3SF</u>) (<u>3SF</u>) (exact) (<u>3SF</u>) (<u>3SF</u>)

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